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Comparative Analysis of Reconciliation Techniques: Bottom-Up, Top-Down, and MinT for Product Forecasting in Retail SMEs

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Abstract **-** Small and Medium Enterprises (SMEs) have experienced rapid growth, contributing approximately 95% to the global economy, 60% to global employment, and 50% to global GDP. This growth is accompanied by significant challenges, with approximately 70% of SMEs failing within the first three years, primarily due to poor inventory management. It emphasizes the crucial role of accurate demand forecasting for SMEs, particularly in the retail sector, where time series at various levels of hierarchical structure exhibit different scales and display diverse patterns. However, most existing research on demand forecasting for SMEs focuses on a single hierarchical level—either bottom, middle, or top—without addressing the entire hierarchy. The research sought to address this gap by forecasting across all hierarchical levels and evaluating different reconciliation techniques to generate coherent and accurate forecasts for multiple products in retail SMEs. The ETS state space model was used as the base forecasting model. This model was widely recognized as a benchmark in forecasting competitions. The reconciliation methods assessed were Bottom-Up, Top-Down based on historical proportions (average proportions), Top-Down based on forecast proportions, and Minimum Trace (MinT) (Ordinary Least Squares (OLS), OLS Non-Negative (OLS Non-Neg), Weighted Least Squares (WLS), and WLS Non-Negative (WLS Non-Neg)). The evaluation results show that the OLS Non-Negative method, with an average SMAPE value of 35.335%, produces more accurate reconciliation than other methods. In addition, this method also outperforms the base model with an increase in accuracy of 13%.

Keywords: Comparative Analysis, Reconciliation Techniques, Bottom-Up, Top-Down, Minimum Trace, Product Forecasting, Retail Small and Medium Enterprises (SMEs)

I. INTRODUCTION

Small and medium-sized enterprises (SMEs) have grown rapidly in recent years. According to World Trade Organization (2022), in 2022, SMEs made up about 95% of the world's business population, employed around 60% of the global workforce, and contributed approximately 50% to the global Gross Domestic Product (GDP). According to Bayraktar and Algan (2019), 20% of SMEs fail in the first year, another 20% in the second year, and about 50% do not survive beyond the first five years. That statement aligns with the findings of Carazas et al. (2020) that 70% to 80% of SMEs fail in the first three years due to poor inventory management. Effective demand forecasting, a critical aspect of inventory management, allows SMEs to estimate customer demand, maintain efficient stock levels, minimize lost sales, and maximize profits (Fildes et al., 2022). According to Saleem et al. (2020), using forecasting models positively correlates with improving SME performance. However, many SMEs still rely on intuition to estimate sales due to limited resources such as data, hardware, software, costs, and knowledge. This fact makes demand forecasting research tailored to SMEs' needs critical.

Making product estimates for SMEs, especially in the SME retail sector, is challenging. The increasing number of items in the retail inventory system makes accurate forecasting increasingly complex due to the variety of demand patterns shown by hundreds or thousands of Stock Keeping Units (SKUs). Another problem arises due to the fact that each SKU usually consists of a particular hierarchical structure (Fildes et al., 2022). Time series at different levels of hierarchical structure have different scales and show very diverse patterns. Time series at the bottom of the hierarchical structure tend to have random variations or fluctuations (noisy) and often show irregular observation patterns (intermittent), making them more challenging to model and predict. Meanwhile, higher-level time series are usually much smoother and easier to predict.

Most demand forecasting research on SMEs only focuses on one hierarchical level: bottom, middle, or top. Several previous studies have tried to overcome intermittent problems at the lowest level using special models to handle intermittent patterns, such as Syntetos Boylan Approximation (SBA), Aggregate Disaggregate Intermittent Demand Approach (ADIDA), and Croston (Chatzipanagioti, 2018). Others focus on clustering time series at mid-level aggregation using clustering (Purnamasari et al., 2023) or based on shared attributes (Isnaini & Sudiarso, 2018). Other previous studies also try to forecast at the highest level of the hierarchy by aggregating the totals of many time series at once (Angulo-Baca et al., 2020; Fahrudin et al., 2022; Kolade et al., 2019; Pratama et al., 2022). These approaches, of course, have their respective advantages. Forecasting at the lowest level of the hierarchy allows the model to capture the entire time series pattern without any information being lost due to aggregation, and forecasting at the middle level of the hierarchy uses clustering techniques, allowing for more flexible model selection options according to the type of characteristics of the time series in each cluster. Meanwhile, forecasting at the top hierarchical level can reduce the complexity of the forecasting model because it only requires one model. However, a model is needed to produce consistent forecasts at various levels of the product hierarchy while maintaining the coherence of forecasting results to make optimal decisions. It is doubtful that this aggregation constraint will be satisfied if the estimates on each series in the hierarchical structure are generated independently. Furthermore, forecasting methods on data with hierarchical patterns should utilize the relationships between series at each hierarchical level (Oliveira & Ramos, 2019). Therefore, the research aims to perform forecasting at all levels of the product hierarchy in SMEs by leveraging the interrelationships between time series through hierarchical reconciliation techniques.

Methods for producing coherent forecasts at all hierarchical levels are generally known as reconciliation methods. The simplest reconciliation methods are the Top-Down and Bottom-Up (Wickramasuriya et al., 2019). The Bottom-Up approach involves making base forecasts on all-time series at the lowest level and aggregating these base forecasts to a higher level (Bertani et al., 2021). The main advantage of this approach is that no information is lost due to aggregation because forecasting is done at the lowest level. The weakness of this method is the potential for the accumulation of errors due to aggregation at higher levels. Meanwhile, the Top-Down approach is carried out by making base forecasts at the highest hierarchical level and disaggregating these base forecasts to the lowest hierarchical level (Anderer & Li, 2022). The limitation of the Top-Down approach, which uses historical proportions, tends to produce less accurate forecasts at lower hierarchical levels.

Empirical studies comparing the performance of Bottom-Up and Top-Down methods provide varying results regarding the choice between Bottom-Up and Top-Down (Athanasopoulos et al., 2024). Recent research in this area addresses these issues using a two-stage approach. In the initial stage, forecasting is carried out for all series at all hierarchical levels. Next, a regression model combines these base forecasts to produce consistent forecasts. According to Athanasopoulos et al. (2009) and Hyndman et al. (2011), the Ordinary Least Squares (OLS) estimator method demonstrates the success of their approach compared to traditional methods. Then, Generalized Least Squares are proposed to improve forecasting accuracy by considering as much variation and interrelationships between errors in the base forecast as possible. However, this approach cannot guarantee that the forecasts produced are non-negative. This weakness becomes a severe problem in inherently non-negative applications like sales data. This problem is later answered by the research of Wickramasuriya et al. (2020) through a quadratic programming approach by applying non-negative constraints.

The selection of a suitable model to forecast multiple series simultaneously involves various options. While machine learning and deep learning models excel at capturing non-linear relationships between input and target variables, they are computationally intensive and heavily reliant on extensive feature engineering. These approaches may not be the best for small and medium-sized businesses operating in environments with limited resources. In such cases, the exponential smoothing model serves as a more practical choice for forecasting. This model offers the advantage of lower computational costs and reduces dependence on complex features of engineering while also striking a balance between accuracy and model complexity. Additionally, the exponential smoothing approach has shown strong performance in a wide range of time series (Panigrahi et al., 2021; Rosenblad, 2021). According to Barrow et al. (2020), the exponential smoothing family model emerges as the most commonly used model in their survey. It is utilized approximately a third of the time, compared to more complex models, which are employed only about 10% of the time.

In the research, an empirical study is conducted using sales data from Funan Mart, a pseudonym for one of the SMEs in Indonesia. The exponential smoothing model is used as the base forecasting model, while the Bottom-Up, Top-Down, and Minimum Trace (MinT) reconciliation models are used as reconciliation comparison methods. The research aims to investigate and gain a deeper understanding of various reconciliation techniques in hierarchical forecasting to produce accurate and coherent multiple-product forecasts in retail SMEs. The research results are expected to provide an in-depth view of the selection of hierarchical levels, the use of reconciliation techniques, and their impact on forecasting accuracy at each hierarchy level.

II. METHODS

A hierarchical time series is a type of multivariate time series in which each time series entity is related to each other through a particular hierarchical structure (Wickramasuriya et al., 2019). An illustration of this structure is in Figure 1. The top of the hierarchy in Figure 1 represents the total number of all-time series contained in the hierarchical structure. The top of the hierarchy is the total of the time series divided by a particular parameter, such as geographic location, product category, or store ID. This structure is then subdivided into more detailed levels using similar methods until it reaches the lowest time series level. In Figure 1, the highest-level time series can represent a retail organization's total sales, the middle-level time series A and B can represent the total sales in each SKU category sold, and the lowest-level time series AA, AB, BA, and BB can reflect the total sales of each SKU in each category being sold.

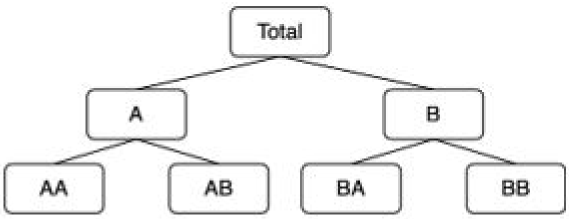


Figure 1 Illustration of Hierarchical Time Series Structure

In particular, following the notation in the research of Wickramasuriya et al. (2019), a set of N time series of length T has a hierarchical structure, where $Y_t \in R^N$ is a vector containing all observations from each time series at time t, and $b_t \in R^M$ is a vector containing observations from M lower-level time series at time t. Then, the hierarchical structure of the time series can be represented in matrix form in Equation (1). The $S \in R^{NxM}$ is a covariance matrix that describes the time series structure from the lowest to the highest level.

$$y_t = Sb_t. \quad (1)$$

The forecasting model used as the base forecast in the research is the exponential smoothing model. Exponential smoothing requires decomposing a time series into three components: trend, seasonal, and error components. Various combinations of these components result in different combinations of exponential smoothing models. In the framework described by Hyndman et al. (2002), an automatic model selection approach is proposed to select the exponential smoothing model that best suits the time series data. This process is carried out through two main stages: maximizing the likelihood for each exponential smoothing model and selecting the model that produces the lowest Akaike's Information Criterion (AIC). This research forecasts at all hierarchical levels using the framework proposed by Hyndman et al. (2002). In the M5 forecasting competition, the exponential smoothing model selection framework is used as the baseline model (Makridakis et al., 2022). This model outperforms the other 50 benchmark models, with only 415 out of 5,507 participating teams (equivalent to 7.5%) surpassing its performance.

In general, predictions produced by base models tend to be incoherent. Therefore, additional methods are needed to ensure the coherence of the time series forecasting results. The most commonly used reconciliation method is linear reconciliation (Wickramasuriya et al., 2019). If $\hat{y}_{t+h}$ is defined as a multi-horizon base forecast over h-steps of the entire time series in the hierarchy and S as a summing matrix describing the hierarchical structure, the linear reconciliation can be formulated in Equation (2). The matrix $P \in R^{MxN}$ serves as the optimal mapping matrix, reconciling the base forecast ($\hat{y}_{t+h}$) with the reconciled forecast ($\tilde{y}_{t+h}$) as shown in Equation (2).

$$\tilde{y}_{t+h} = SP\hat{y}_{t+h}. \quad (2)$$

The Bottom-Up approach involves making base forecasts for all-time series at the lowest level and aggregating these base forecasts to higher-level hierarchies. In this approach, the matrix P is mapped in Equation (3). In accordance with the notation used by Wickramasuriya et al. (2019), N states the total number of time series in a hierarchical structure, while M is the number of time series at the lowest level.

$$P = [0_{M\times(M-N)} | I_M]. \quad (3)$$

The $0_{M\times(M-N)}$ is the null matrix of $M\times(N-M)$, and l_M is the $M\times M$ identity matrix. The role of the P matrix here is to extract forecasts at the lower level, which are summed by the S summation matrix to produce forecast reconciliation $\tilde{y}_{t+h}$ for all levels of the hierarchy.

The Top-Down approach involves forecasting the time series at the highest hierarchical level and disaggregating the forecasting results to the lowest hierarchical level. In this approach, the P matrix is mapped in Equation (4). In this context,

$p = [p_1, p_2, \dots p_M]^T$ represents the set of proportions corresponding to a series at the lower level. The role of the P matrix in the Top-Down approach is to distribute the predictions at the top level into predictions for the series at the bottom level. There are several variants of the Top-Down approach. The first approach is Top-Down, based on average historical proportions. In this approach, the proportion of the matrix P is represented in Equation (5). Here, $y_{j,t}$ denotes the historical data value of the lower-level series j in period t. Each proportion P_j reflects the average historical proportion of the lower-level series $\{y_{j,t}\}$ over the period $t = 1, \cdots, n$ compared to the aggregate total $\{y_t\}$. The second approach is the proportions of historical averages. In this approach, the matrix P can be mapped in Equation (6). Each proportion, denoted as P_j, reflects the historical average value of a specific lower-level series, represented by $\{y_{j,t}\}$, compared to the overall average value of the aggregate series $\{y_t\}$.

$$P = [p \,|\, 0_{M\times(N-1)}], \tag{4}$$

$$P_j = \frac{1}{n}\sum_{t=1}^{n}\frac{y_{j,t}}{y_t}, \tag{5}$$

$$P_j = \sum_{t=1}^{n}\frac{y_{j,t}}{n} \Big/ \sum_{t=1}^{n}\frac{y_t}{n}. \tag{6}$$

Top-Down approaches using historical proportions for disaggregation often produce less accurate forecasts at lower hierarchy levels than Bottom-Up approaches. It is because historical proportions do not account for potential changes in proportions over time (Athanasopoulos et al., 2009). For example, in a retail context, beverage products have a different time series pattern from household products. Due to aggregation effects, a Top-Down approach based on historical proportions cannot capture such patterns. To address this issue, proportions based on historical forecasts can be used. This approach creates independent forecasts for each series across all hierarchy levels. At each hierarchical level, the proportion of each individual forecast to the aggregate total is calculated using Equation (7). The $\hat{y}_{j,h}^{(l)}$ is the h-step ahead forecast for the series corresponding to nodes that are l levels above j. Then, $\hat{S}_{j,h}^{(l)}$ is the number of initial h-step-ahead predictions under a node that is l levels above node j and directly connected to that node. These proportions are then used to distribute forecasts from top to bottom. This process is repeated for each node from the top to the bottom level in the hierarchy.

$$P_j = \prod_{l=0}^{n-1}\frac{\hat{y}_{j,h}^{(l)}}{\hat{S}_{j,h}^{(l+1)}}. \tag{7}$$

Top-Down and Bottom-Up reconciliation methods suffer from information loss because forecasting is only based on forecasts at the top or lowest level of the hierarchy. Previous research by Wickramasuriya et al. (2019) proposes an approach that minimizes the trace of the covariance matrix P of the reconciliation forecast error to utilize information from all levels in the hierarchy.

Assuming that the base forecasts are unbiased, the forecast error at horizon h is defined as $\hat{e}(h) = y_{t+h} - \hat{y}_{t+h}$. Under this assumption, the expected value of the forecast error is zero, i.e., $E[\hat{e}(h)] = 0$. Consequently, the expected value of the base forecast is equal to the expected value of the actual future observation, expressed as $E[y_{t+h}] = E[\hat{y}_{t+h}]$. Let $\hat{b}_{t+h}$ denote the base forecasts at the lowest (bottom) level. If these forecasts are unbiased, it is $E[y_{t+h}] = S\beta_{t+h}$, where β_{t+h} represents the true expected values of the bottom-level series. According to the linear reconciliation in Equation (2), reconciled forecasts are computed as $\tilde{y}_{t+h} = SP\hat{y}_{t+h}$, where S is the summing matrix and P is the reconciliation (or projection) matrix. Taking the expectation of both sides and substituting the previous relationships yields $E[\tilde{y}_{t+h}] = SPE[\hat{y}_{t+h}] = SPS\beta_{t+h}$. To ensure that the reconciled forecasts remain unbiased, it is $E[\tilde{y}_{t+h}] = E[y_{t+h}] = S\beta_{t+h}$, and the following condition must hold: $PS = I_N$, where $I_N \in R^{N\times N}$. According to Wickramasuriya et al. (2019), the primary objective of the MinT reconciliation method is to determine the matrix P that satisfies the equation $PS = I_N$ because minimizing the trace of the covariance matrix P is equivalent to minimizing the sum of the variances of the reconciled forecast errors. The solution to this problem is described in Equation (8). The W_h represents the covariance matrix of the base forecast errors at horizon h. Because the base forecasting error cannot be calculated in advance, the value of W_h must be estimated.

$$P = (S'W_h{}^{-1}S)^{-1}S'W_h{}^{-1}. \tag{8}$$

In OLS, the value is $W_h = I_N$. This method is the same as the OLS estimator model, so the value of the matrix P can be seen in Equation (9). Then, in WLS$_s$, the value is $W_h = \Lambda$, where Λ is a diagonal matrix with value $S1$. Then, S is the summing matrix and $1 \in R^{M\times 1}$ is the unit vector. This method assigns weight to the top-level series M and 1 for the weight of the bottom-level series. Therefore, this method is called WLS$_s$ (Weighted Least Squares applying structural scaling).

$$P = (S'I_N^{-1}S)^{-1}S'I_N^{-1} = S'S^{-1}S'. \tag{9}$$

The main issue with the MinT reconciliation approach is that it does not ensure that the reconciled forecast will always be non-negative. It can be a significant problem in sales forecasting, where the target variable is inherently non-negative. The forecast reconciliation is redefined using quadratic programming, which imposes a non-negative constraint to solve this problem. It ensures that the final reconciled forecast will have a non-negative value.

Several metrics are used to evaluate the performance of various forecasting models. These metrics include Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE). The formulation of MAE and RMSE is defined in Equations (10) and (11). Next, the formulation of MAPE is in Equation (12). The n is the number of observations, y_t is the actual value, and $\hat{y}_t$ is the predicted value at time t.

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t|, \tag{10}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(|y_t - \hat{y}_t|)^2}, \tag{11}$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|100\frac{y_t - \hat{y}_t}{y_t}\right|. \tag{12}$$

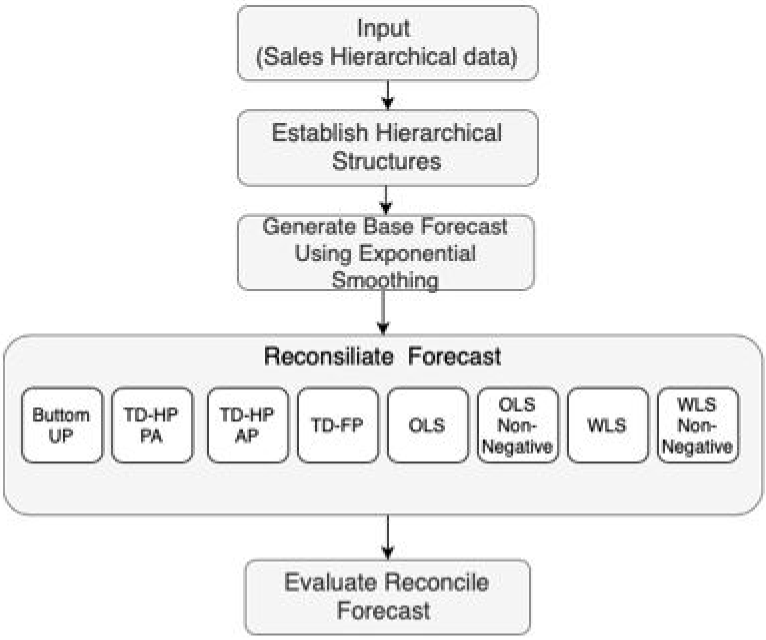
The main problem with using MAPE is that when (yt) has a value of zero, the divisor becomes zero, and the division result becomes undefined. Symmetric Mean Absolute Percentage Error (SMAPE) overcomes this problem by calculating the prediction error relative to the average of the actual value and the predicted value, not just the actual value. The smaller the SMAPE value, the better the forecasting results. SMAPE is shown in Equation (13). Interpretation of MAPE and SMAPE can be seen in Table 1.

$$SMAPE = \frac{100\%}{n}\sum_{t=1}^{n}\frac{|y_t - \hat{y}_t|}{|y_t| + |\hat{y}_t|} \tag{13}$$

Figure 2 illustrates the complete flow of the research. The data are from sales database of Funan Mart, which was processed through daily aggregation from November 2022 to September 2023, encompassing a total of 307 days. Table 2 provides a detailed description of the dataset.

Table 1 Interpretation of Mean Absolute Percentage Error (MAPE) and Symmetric Mean Absolute Percentage Error (SMAPE)

MAPE Value	SMAPE Value	Predictive Performance Evaluation
< 10%	< 10%	Very accurate forecast
10–20%	10–20%	Good forecast
20–50%	20–50%	Reasonable forecast
> 50%	> 50%	Inaccurate forecast



Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

Figure 2 Research Flow

Table 2 Dataset Description

Column	Data Type	Total Records
Product Code	object	293,185
Product Name	object	293,185
Transaction Time	datetime64[ns]	293,185
Quantity	int64	293,185
Category Code	object	293,185

The number of unique Product Codes in the dataset is 955. After further analysis, it is found that ten products are not sold at all during the period from November 14, 2022, to September 16, 2023. Hence, these products are removed. Furthermore, 406 products are only sold in less than 10 days out of 307 days. These products are removed before forming the hierarchical structure. An example of a removed intermittent time series can be seen in the Appendix. The main reason for removing products with very sporadic intermittent patterns, especially in a hierarchical context, is that they can affect the quality of the aggregation calculation. In a hierarchical context, the middle and upper levels are the aggregates of the products at the lower levels. Products with a very intermittent pattern whose sales value is mostly zero but are punctuated by sporadic sales spikes can cause aggregate value distortion. These irregular spikes can introduce bias in the trend or demand patterns at higher hierarchical levels. The data's hierarchical structure is divided into three levels. Table 3 shows the entire time series at each hierarchical level. Level 1 of the hierarchy represents the total sales. Level 2 represents sales by product category, and level 3 represents sales on each SKU. This hierarchy is formed using the 'hierarchical forecast' library.

Table 3 Number of Series in Each Hierarchy

Level	Total Series in Each Level
Level 1	1
Level 2	14
Level 3	539
All Series	**554**

There are 13 recorded days with zero sales. These days can be identified through review at the highest hierarchical level of the data. The researchers perform imputation to handle days with zero sales using the time interpolation method from the Pandas library. The data are then divided into a training set and a test set, with the training set consisting of 293 days and the remaining 14 days used as the test set. Training is carried out using the exponential smoothing model with the Stats Forecast library to produce base forecasts. The base forecasting results are then reconciled using Bottom-Up, Top-Down historical average proportions denoted as TD-HP-AP, Top-Down based on historical proportion averages denoted as TD-HP-PA, Top-Down based forecast proportions denoted as TD-FP, MinT, and Non-Negative MinT techniques. The reconciliation results are then compared using SMAPE, MAE, and RMSE metrics.

III. RESULTS AND DISCUSSIONS

Table 4 shows the performance comparison of each reconciliation model based on MAE at various hierarchy levels. Table 5 presents the performance of each model based on RMSE. Finally, Table 6 compares the models' performance using SMAPE.

Table 4 Mean Absolute Error Values at Each Hierarchy Level

Model	Aggregation Level			All Levels
	Top-Level	Middle Level	Bottom Level	
Base	66.629	12.326	1.091	1.494
Bottom-UP	81.478	12.526	1.091	1.526
TD-HP-PA	66.629	14.119	1.178	1.623
TD-HP-AP	66.629	13.691	1.186	1.620
TD-FP	66.629	12.241	**1.074**	**1.474**
OLS	**66.604**	12.465	1.107	1.512
OLS Non-Neg	66.654	12.390	**1.074**	1.479
WLS	71.372	**12.165**	1.096	1.503
WLS Non-Neg	72.885	12.253	1.084	1.496

Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

All reconciliation models show high accuracy at the top hierarchical level, with SMAPE values ranging from 6.239 to 7.875, MAE values ranging from 66.604 to 81.478, and RMSE values ranging from 80.503 to 106.526. At this hierarchical level, all models based on the Top-Down approach produce consistent and similar predictions, with MAE values of 66.629, RMSE of 80.789, and MAPE of 6.239. This uniformity of results is due to the characteristics of the Top-Down approach, which only uses the base forecast results from the top level of the hierarchy as a base for distributing values to the levels below. The OLS-based reconciliation model also performs well and is comparable to the Top-Down approach, as seen from the MAE, RMSE, and SMAPE values, which are not significantly different. It indicates the OLS approach can provide effective reconciliation at a high-level hierarchy without losing significant accuracy.

On the other hand, the Bottom-Up reconciliation model shows the worst performance compared to other reconciliation methods, with an MAE value of 81.478, RMSE of 106.526, and SMAPE of 7.875. The significant difference between the MAE and RMSE values in the Bottom-Up approach at the high hierarchy level indicates significant, uneven errors at the top level. This phenomenon is caused by the accumulation of errors due to the data aggregation process from lower to higher levels of the hierarchy. In addition, at this level of the hierarchy, the Weighted Least Squares (WLS) reconciliation approach that gives greater weight to the top level produces less satisfactory performance than other methods. It is indicated by the MAE value of 71.372, RMSE of 86.221, and SMAPE of 6.859, which are quite different from other models. These results indicate that giving suboptimal weights to the top level of the hierarchy can increase the model's

Table 5 Root Mean Squared Error (RMSE) Values at Each Hierarchy Level

Model	Aggregation Level			All Level
	Top-Level	Middle Level	Bottom Level	
Base	80.789	15.373	1.434	1.929
Bottom-UP	106.526	16.109	1.434	1.994
TD-HP-PA	80.789	17.302	1.551	2.092
TD-HP-AP	80.789	17.024	1.563	2.097
TD-FP	80.789	**15.356**	**1.424**	**1.919**
OLS	**80.503**	15.574	1.454	1.954
OLS Non-Neg	80.524	15.514	1.436	1.935
WLS	86.221	15.387	1.441	1.946
WLS Non-Neg	88.598	15.480	**1.433**	1.946

Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

Table 6 Symmetric Mean Absolute Percentage Error (SMAPE) Values at Each Hierarchy Level

Model	Aggregation Level			Average
	Top-Level	Middle Level	Bottom Level	
Base	**6.239**	**32.727**	84.290	41.085
Bottom-UP	7.875	33.499	84.290	41.888
TD-HP-PA	**6.239**	35.128	88.184	43.184
TD-HP-AP	**6.239**	34.960	88.223	43.141
TD-FP	**6.239**	32.842	84.439	41.173
OLS	6.244	38.643	87.510	44.132
OLS Non-Neg	6.250	33.064	**66.691**	**35.335**
WLS	6.859	32.960	87.379	42.399
WLS Non-Neg	7.025	33.147	74.632	38.268

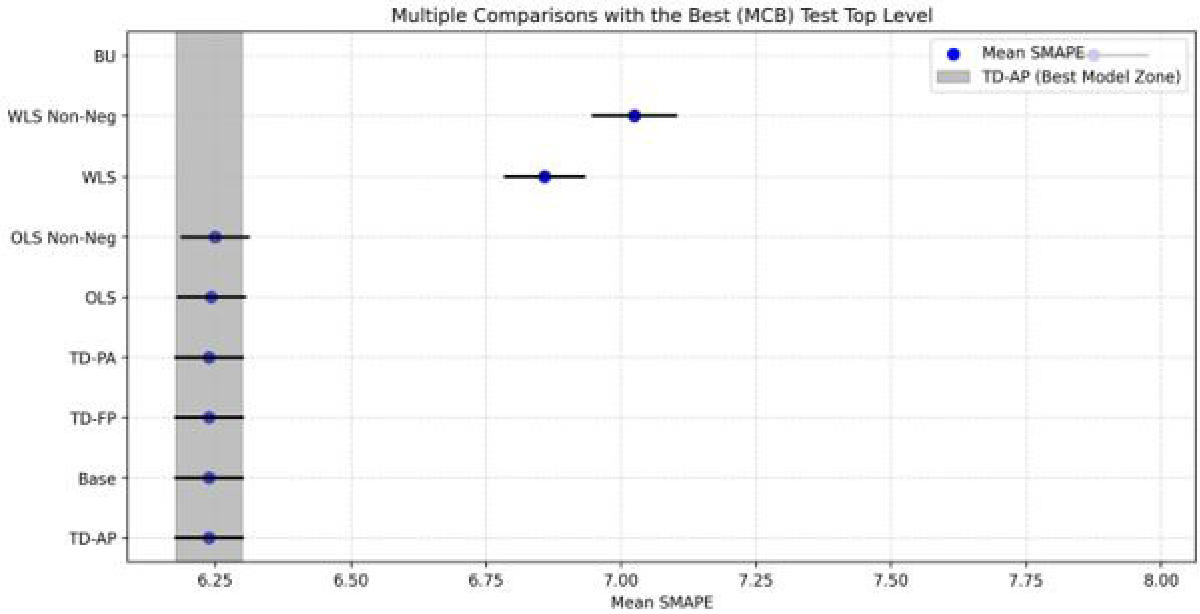
Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

sensitivity to errors that occur at the top level, thereby worsening the prediction accuracy at the top level. Therefore, more strategic weight settings are needed to increase the effectiveness of the WLS approach in maintaining accuracy at upper hierarchical levels.

Figure 3 displays the results of the accuracy rankings calculated using the Multiple Comparisons with the Best (MCB) test to evaluate the significance of the findings at the top hierarchical level to confirm these findings. This method has been widely used in forecasting literature. The MCB test evaluates the performance of methods by comparing the average of their rankings, using critical differences determined through confidence intervals (Koutsandreas et al., 2022). The MCB test is applied at the top hierarchical level, and the results are presented for each forecasting method. In each panel of Figure 3, methods not in the gray area (representing the best-ranked method) are considered to have significantly worse performance than the best method. The results of this visual analysis show that Top-Down reconciliation methods such as TD-HP-PA, TD-HP-AP, and TD-FP, as well as OLS-based methods (including OLS with non-negative restrictions) and direct forecasting from the base model, produce identical performance. In contrast, Bottom-Up reconciliation shows the lowest performance, and WLS does, too. Methods that do not overlap the gray area, showing the confidence interval of the best method, are considered to have significantly lower performance than the best method.

At the middle hierarchy level, the forecasting results are greatly influenced by the type of reconciliation model used. In the Top-Down approach, the forecast at this level is obtained through a disaggregation process from the forecast results at the upper hierarchy level. The TD-HP-PA reconciliation method uses a disaggregation method based on the historical proportion average. This approach produces MAE values of 14.119, RMSE of 17.302, and SMAPE of 35.128. Meanwhile, the TD-HP-AP reconciliation model uses the historical average proportion method, which gives slightly better results, with MAE values of 13.691, RMSE of 17.024, and SMAPE of 34.960. In the Top-Down forecast proportion approach, the proportions used are taken from the latest forecast results (forecast proportion), not historical data. This approach allows the model to be more responsive to the latest data patterns than historical proportion-based methods that rely only on past information. It can be confirmed by the MAE value of 12.241, RMSE of 15.356, and SMAPE of 32.842, which is better than the two previous Top-Down reconciliations.

On the other hand, in the OLS and WLS reconciliation models based on structural scaling, the forecast results at the middle hierarchy level are influenced by all base forecasts at all levels in the hierarchy. The main difference between OLS and WLS lies in how they consider the hierarchical structure in the reconciliation process. In the OLS method, all elements in the reconciliation are given the same weight without considering differences in scale or structure between hierarchy levels. In contrast, weights



Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

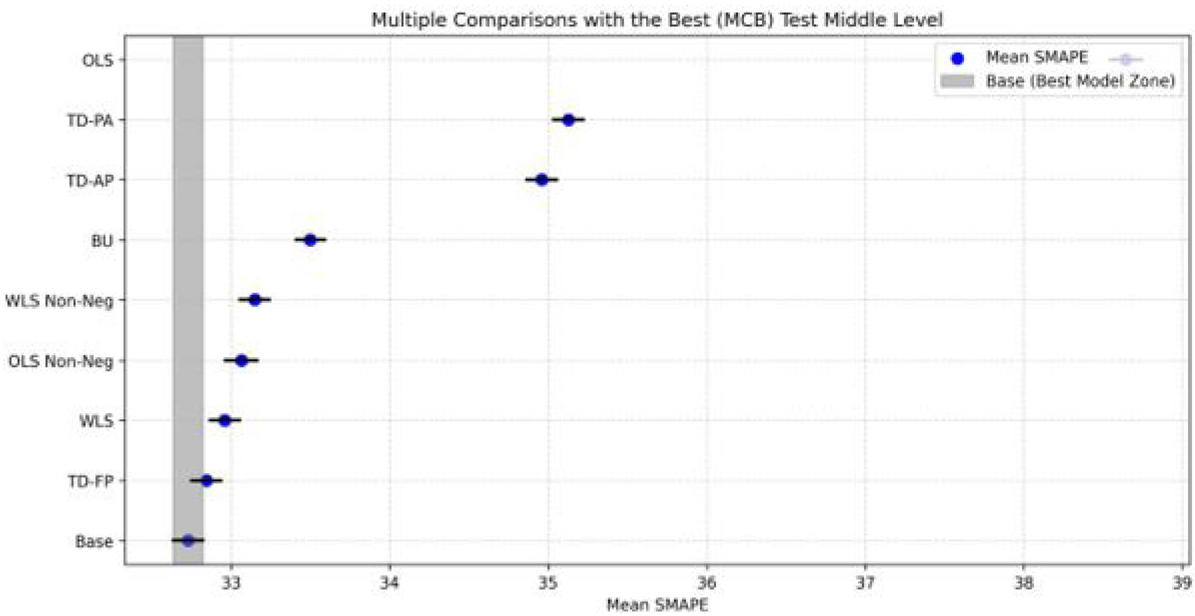
Figure 3 Multiple Comparisons with the Best (MCB) Test
with a Confidence Level of 0.95 at the Top Hierarchical Level

are given according to the hierarchical structure in the WLS method with a structural scaling approach. In contrast, levels with larger data scales or influences are given higher weights.

The forecasting results at the middle hierarchy level are MAE of 12.465, RMSE of 15.574, and SMAPE of 38.643. The results are slightly worse than WLS, with MAE of 12.165, RMSE of 15.387, and SMAPE of 32.960. Although WLS with structural scaling offers greater flexibility in handling scale imbalances between levels, this approach requires careful weight settings. Inappropriate weights can increase the model's sensitivity to errors at a certain level, especially if there are levels with high data variability. As a result, the reconciliation results can be less stable compared to the OLS approach, which gives uniform weight to each element, as seen at the top level of the hierarchy, whereas WLS reconciliation produces poor results. Overall, the model accuracy is worse at the middle hierarchy level than at the top hierarchy level, with a range of SMAPE values between 32.727 and 38.643, MAE values from 12.165 to 14.119, and RMSE values from 17.302. Figure 4 shows the results of the MCB test to assess further the performance of each model at the middle hierarchy level. From the results of the MCB test, it is found that the base model is the most accurate, while the Top-Down forecast proportion reconciliation model is the most accurate. Methods that do not overlap the gray area, showing the confidence interval of the best method, are considered to have significantly lower performance than the best method.

The lowest level of the hierarchy shows the lowest forecasting accuracy compared to other levels. At this level, the SMAPE value ranges between 66.691 and 88.223, the MAE value between 1.074 and 1.186, and the RMSE value between 1.424 and 1.563. At this level, the Bottom-Up reconciliation results are identical to the base model results, with an MAE value of 1.091, RMSE of 1.434, and SMAPE of 84.290. It can be explained by the fact that the Bottom-Up method only accumulates values from the lowest level of the hierarchy to the top, so it does not change the base prediction results at this level. As previously expected, the Top-Down approach based on historical proportions produces worse accuracy at the lowest level of the hierarchy.

The TD-HP-PA model using the disaggregation method based on the historical proportion average produces an MAE value of 1.178, an RMSE of 1.551, and a SMAPE of 88.184. Meanwhile, the TD-HP-AP model, which uses the historical average proportion method, shows similar performance, with an MAE value of 1.186, an RMSE of 1.563, and an SMAPE of 88.223. Both are proven to provide the worst forecasting results at this hierarchy level because both methods rely entirely on historical proportions without considering current patterns that may be more relevant. In contrast, the Top-Down forecast proportion approach, which uses proportions based on current forecast results (forecast proportions) instead of historical data, shows better performance



Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

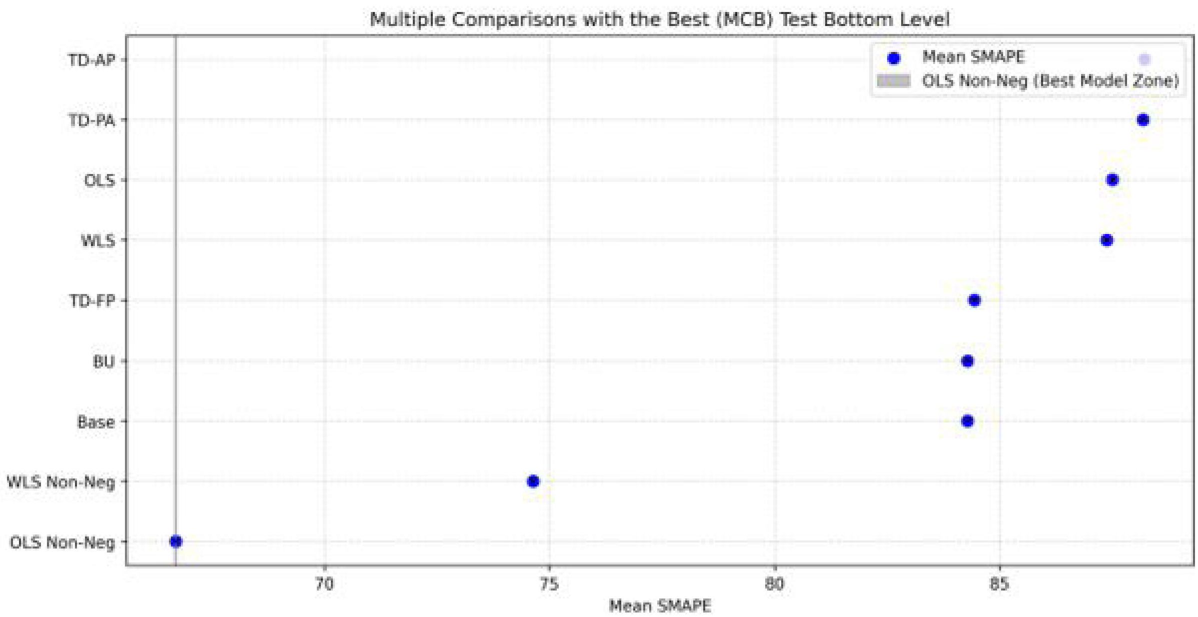
Figure 4 Multiple Comparisons with the Best (MCB) Test
with a Confidence Level of 0.95 at the Middle Hierarchy Level

than the other two Top-Down methods. This approach produces an MAE value of 1.074, an RMSE of 1.424, and an SMAPE of 84.439, almost equivalent to the baseline model. The low accuracy of the base model at the lowest level of the hierarchy also affects the results of the OLS and WLS-based reconciliation.

Reconciliation with the OLS method produces an MAE value of 1.107, RMSE of 1.454, and SMAPE of 87.510, while the WLS method shows an MAE value of 1.096, RMSE of 1.441, and SMAPE of 87.379. This poor performance can be explained by the dependence of both methods on the results of the base forecasts, which, at the lowest level of the hierarchy, have low accuracy. Poor base forecast results also impact the OLS and WLS reconciliation results at the middle level of the hierarchy, considering that these reconciliation methods consider the contribution of all levels in the hierarchy. However, using non-negative constraints in the OLS and WLS methods has improved reconciliation accuracy at the lowest level of the hierarchy.

By applying these restrictions, the reconciliation results show an increase in accuracy compared to the OLS and WLS methods without restrictions, surpassing the base model results. In addition, the use of non-negative constraints also contributes positively to the accuracy of reconciliation at the middle hierarchy level, making this approach more effective in overcoming the weaknesses of the base model, which has low accuracy at the lowest hierarchy level. Figure 5 shows the results of the MCB test at the lowest hierarchy level to assess further each model's performance at the middle hierarchy level. From the results of the MCB test, the OLS non-negative reconciliation model is the most accurate, followed by the WLS non-negative reconciliation model. Methods that do not overlap the gray area, showing the confidence interval of the best method, are considered to have significantly lower performance than the best method.

Although the accuracy of each hierarchical reconciliation model has been described separately at the upper, middle, and lower levels, it is important to assess the overall effectiveness of the models because hierarchies are interdependent. A model that appears accurate at one level may produce inconsistencies or error propagation when connected to other levels. In addition, the main purpose of hierarchical reconciliation is to ensure the alignment of predictions across levels, so model performance is not only assessed based on local accuracy (per hierarchy level) but also the ability to maintain data integrity globally. Therefore, an assessment covering all levels of the hierarchy needs to be carried out to ensure that the model's performance is optimal for all levels of aggregation. The Top-Down approach based on historical proportions gave the worst reconciliation results compared to other models. In the TD-HP-PA model, the disaggregation method is carried out based on the average of historical proportions, resulting in an MAE value of 1.623, RMSE of 2.092, and an average SMAPE of 43.184. Meanwhile, the TD-HP-AP model, which uses the average historical proportions, produces an MAE value of 2.097, an RMSE of 2.097, and an average SMAPE of 43.141. These poor results occur because both methods rely entirely on historical information, which cannot capture current patterns in

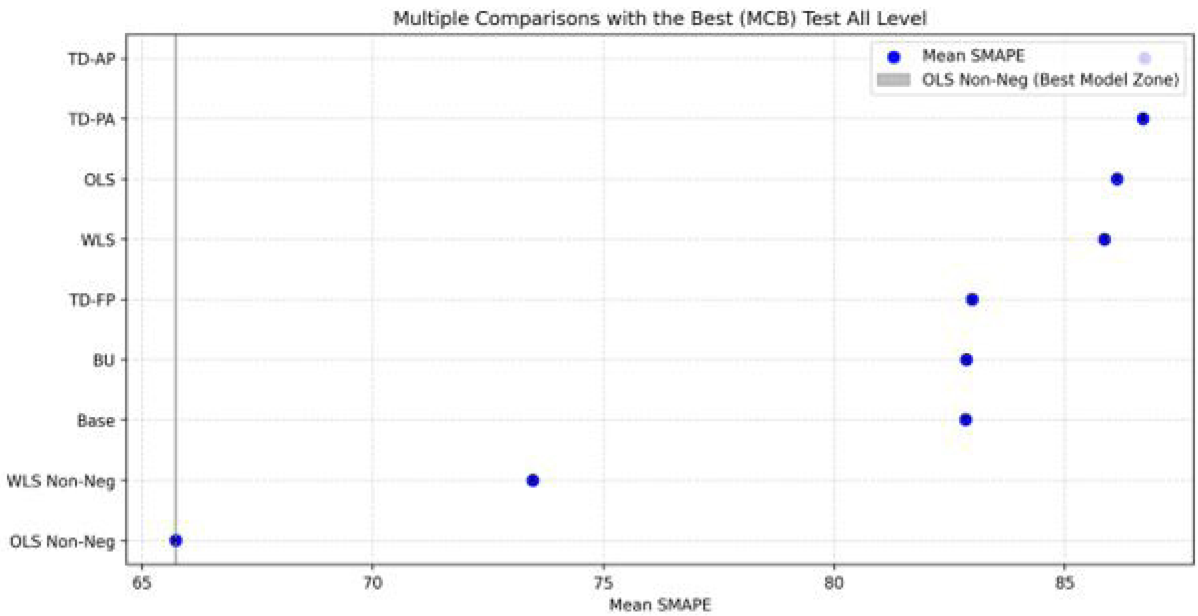


Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

Figure 5 Multiple Comparisons with the Best (MCB) Test with a Confidence Level of 0.95 at the Bottom Hierarchy Level the data. In contrast, the Top-Down forecast proportion approach, which uses proportions based on current forecast results (forecast proportions) instead of historical proportions, performs better. This approach makes the model more responsive to current data patterns, resulting in more accurate reconciliations than the two historical proportion-based models. This model produces an MAE value of 1.474, an RMSE of 1.919, and an SMAPE of 41.173, almost equivalent to the forecast results of the baseline model.

In the OLS model, each element in the reconciliation process is given the same weight without considering the scale differences between the hierarchy levels. In contrast, the WLS model uses weights adjusted to the hierarchical structure, where levels with more significant influence in the hierarchy are given higher weights. The poor baseline forecasting results at the lowest level of the hierarchy with MAE values of 1.091, RMSE of 1.434, and SMAPE of 84.290 cause error propagation to all levels of the hierarchy, which negatively impacts the reconciliation performance in both the OLS and WLS models. Applying non-negative constraints to both models successfully improves accuracy, especially at the lowest level of the hierarchy. In the OLS model with non-negative constraints, the MAE value decreases to 1.074, the RMSE to 1.436, and the SMAPE to 66.691. The result shows a significant improvement compared to OLS without non-negative constraints (MAE of 1.107, RMSE of 1.454, and SMAPE of 87.510). The same thing also happens in the WLS model with non-negative constraints, where the MAE value becomes 1.084, RMSE to 1.433, and SMAPE to 74.632. This figure is better than WLS without non-negative constraints, recording an MAE of 1.096, RMSE of 1.441, and SMAPE of 87.379. The increase in accuracy at the lowest level of the hierarchy positively impacts accuracy at all other levels of the hierarchy. Therefore, the OLS and WLS models with non-negative constraints show superior performance compared to other reconciliation models, including the base predictions without reconciliation. Between the two approaches, the OLS model with non-negative constraints performs better than the WLS model. This can be explained by the characteristics of the forecasting problem, which are more significant at the lowest level of the hierarchy, where the prediction accuracy tends to be low. Meanwhile, the middle and upper levels of the hierarchy perform more stable. The use of weights in the WLS model, which considers a hierarchical structure, actually worsens the reconciliation accuracy, especially at the upper levels of the hierarchy, because larger weights are allocated to higher-scale levels without considering the base prediction quality. Thus, the OLS model with non-negative constraints successfully achieves an optimal balance between local (lower level) and global (upper level) accuracy in the hierarchical structure.

Figure 6 presents the results of the MCB analysis at all levels of the hierarchy to confirm this finding. This visual analysis shows that the OLS model with non-negative constraints produces the best reconciliation performance, followed by the WLS model without non-negative constraints. Methods that do not overlap the gray area, showing the confidence interval of the best method, are considered to have



Note: TD-HP-PA: Top-Down Historical Proportion Average, TD-HP-AP: Top-Down Historical Average Proportion, TD-FP: Top-Down Forecast Proportions, OLS: Ordinary Least Squares, OLS Non-Neg: Ordinary Least Squares Non-Negative, WLS: Weighted Least Squares, and WLS Non-Neg: Weighted Least Squares Non-Negative.

Figure 6 Multiple Comparisons with the Best (MCB) Test with a Confidence Level of 0.95 for All Hierarchy Levels significantly lower performance than the best method.

Retail SMEs often face challenges in the form of high levels of demand volatility across various product hierarchy structures, which can complicate stock management and make the right strategic decisions. In this context, applying the OLS model with non-negative constraints provides significant benefits because this model can improve prediction accuracy at the SKU level. A better level of accuracy is essential to support the procurement process and optimal stock replenishment. Thus, SMEs can minimize potential losses caused by overstocking or understocking, ultimately increasing operational efficiency and profitability. As the scale and complexity of SME operations grow, the need to manage data and perform forecasting at various levels of the hierarchy increases, such as from product to category, from category to store, and to region. The OLS model with non-negative constraints has a significant advantage because it can produce consistent predictions across all hierarchy levels. This consistency plays an important role in supporting operational efficiency, such as through more accurate supply chain planning while ensuring alignment of business strategies between local and global levels. In addition, this model enables SMEs to respond to changes in market demand more quickly and effectively, thereby reducing resource waste, improving planning accuracy, and strengthening competitiveness in a dynamic and competitive market.

IV. CONCLUSIONS

The OLS reconciliation model applying non-negative constraints is proven to be a superior and more stable reconciliation method at three levels of product hierarchy based on daily data compared to other reconciliation methods tested. The application of non-negative constraints significantly increases the accuracy at the lowest level of the hierarchy, ultimately contributing to the reconciliation results' stability across all levels. This method produces an average SMAPE value of 35.335%. It performs better than other methods. In addition, the OLS method with non-negative constraints also outperforms the base model with an accuracy increase of 13%, where the base model has a SMAPE value of 41.085%. The research results can be used as a reference in forecasting reconciliation research, especially in the retail SME sector with daily data, where forecasting at the lowest level of the hierarchy is not optimal and produces negative values.

Furthermore, the OLS reconciliation model tested uses a uniform weighting approach at all hierarchy levels. On the other hand, the WLS model assigns weights based on the hierarchical structure, so the WLS forecasting results with structural scaling tend to be less stable than the OLS method. This instability is caused by the characteristics of the forecasting problem, which are more prominent at the lowest level of the hierarchy, where forecast accuracy tends to be low. Meanwhile, the middle and upper levels of the hierarchy show more stable performance.

Future research can consider a more optimal weighting approach. One alternative that can be explored is assigning weights based on forecast variance. It has the potential to produce a more accurate and robust reconciliation model at all levels of the hierarchy.

ACKNOWLEDGEMENT

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AUTHOR CONTRIBUTIONS

Designed the research framework, defined the problem statement, and formulated the methodological approach to evaluate reconciliation techniques for hierarchical time series forecasting, D. H. R.; Communicated, collected, and pre-processed sales data from Funan Mart retail SMEs, ensuring consistency and readiness for forecasting analysis, D. H. R.; Developed custom scripts and adapted existing forecasting libraries to support model implementation and hierarchical reconciliation, D. H. R.; Performed the forecasting simulations, implemented reconciliation methods (Bottom-Up, Top-Down, MinT), and evaluated accuracy using symmetric mean absolute percentage error (SMAPE), D. H. R.; Drafted and revised the manuscript, including abstract, introduction, methodology, results, discussion, and conclusion sections to align with academic standard, D. H. R.; Provided theoretical insights and contributed to the design of the research, R. K. and A. S.; Assisted in interpreting the forecasting results and evaluating the performance of the reconciliation methods, R. K.; Reviewed and edited the manuscript for conceptual clarity, academic rigor, and linguistic precision, R. K. and A. S.; and Supervised the overall research project and provided critical feedback throughout the research process, R. K. and A. S.

DATA AVAILABILITY

The dataset used in the research comes from Funan Mart, a local SME in Indonesia. As this dataset is proprietary, its usage is subject to permission from the company. Researchers who are interested in utilizing this dataset for future studies can contact the corresponding author who can facilitate communication with Funan Mart regarding data access permissions.

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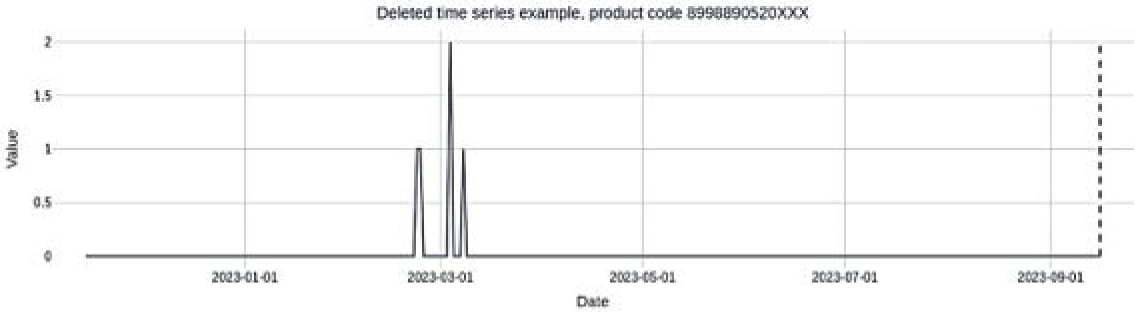
APPENDIX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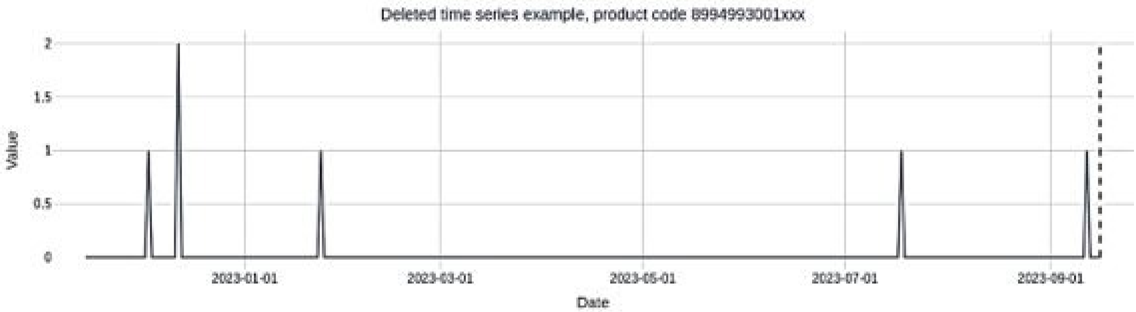
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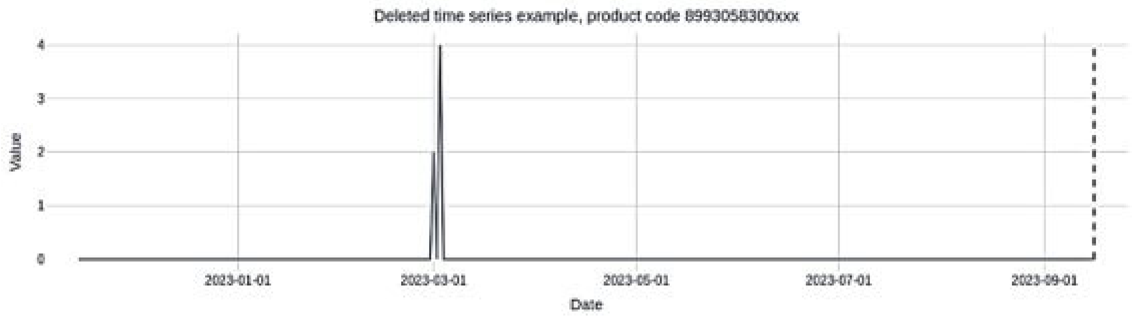
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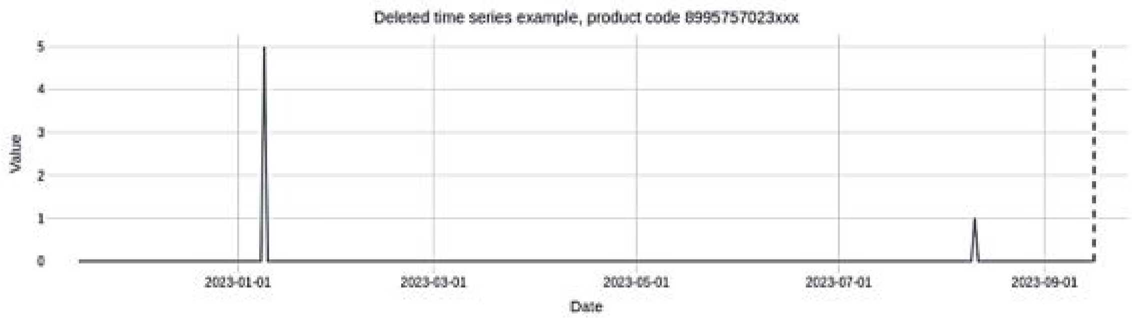
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