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Efficient Computation of Number Fractions from the Square Root of Two Using the A-B Goen Number Function Via the Ivan Newton (in) Series

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Abstract – *The square root number of two is an irrational number. If it is an irrational number, the result cannot be written as a fraction of the numerator and denominator. Fractions that approach the square root value of two have a correlation with Goen's A-B numbers. The regularity of the A-B Goen number sequence can be formulated into the A-B Goen function which is built from the Ivan Newton series. In this research, it can be proven that the A-B Goen function from the Ivan Newton (IN) series is computationally more effective and efficient when compared to the A-B Goen generating function in producing A-B Goen numbers which in infinite sequence will approach the square root value of two.*

Keywords*: Numbers; Functions; Square Root of Two; Irrational Numbers*

I. INTRODUCTION

The definition of an irrational number is a real number that cannot be divided, meaning that the results of the division never stop and never repeat (Lord 2008). Therefore, irrational numbers cannot be expressed as a/b, where a and b are integers and the value of b is not equal to zero (Marshall 2012). So the square root value of two irrational numbers cannot be written in fractional form of the numerator and denominator (Mitchell 2003 and Gel'fand 2023). It is known that there is a pattern of regularity in the form of a correlation between the constituent fractions that are close to the square root value of two and the numbers A Goen and B Goen. The regularity of the A-B Goen number sequence can be formulated into the A-B Goen function which is built from the Ivan Newton (IN) series. In this research, by using the A-B Goen function we can obtain the sequence of A Goen and B Goen numbers. The results obtained from this computational calculation turn out to be more effective and efficient when compared to using the A-B Goen generating function. The A-B Goen function formula to approach the square root value of two can produce the A-B Goen number. The calculation process consists of dividing the number A Goen as the numerator and the number B Goen as the denominator. When compared with the larger sequence of A-B Goen numbers, the result will be closer to the square root value of two.

II. METHODS

2.1 Function A-B Goen with IN Series

The A-B Goen function which is built using the Ivan Newton (IN) Series in the form of a basic equation (Goenawan 2020) is:

$$f(t) = b_0 + b_1 \cdot t + b_2 \cdot \frac{t\cdot(t+1)}{2!} + b_3 \cdot \frac{t\cdot(t+1)\cdot(t+2)}{3!} + \cdots + b_\beta \cdot \frac{\prod_{k=0}^{\beta-1}(t+k)}{\beta.!} \quad (1)$$

By using the IN series interpolation method, the equations for the b_l constants that make it up can be obtained, namely:

$b_0 = f(0), \qquad b_1 = -(f(-1) - b_0)$

and

$$b_l = \left(f(-l) - b_0 - \sum_{j=1}^{l-1} b_j \cdot \frac{\prod_{k=0}^{j-1}(-l+k)}{(j-1)!} \right) \cdot \frac{l!}{\prod_{k=0}^{l-1}(-l+k)} \qquad (2)$$

where the range of values l = 2,3,4….. (positive integer number). From eq.(2) above, a relationship can be obtained to obtain a simpler value of the constant b_l in the IN series using the Pascal's Triangle Number Pattern (Goenawan 2021), namely:

Table 1: Pascal's Triangle Number Pattern

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

From the Pascal's triangle pattern above, the constant equation b_l can be constructed as follows:

$$\begin{aligned}
b_0 &= +f(t=0) \\
b_1 &= -f(t=-1) + b_0 \\
b_2 &= +f(t=-2) - b_0 + 2 \cdot b_1 \\
b_3 &= -f(t=-3) + b_0 - 3 \cdot b_1 + 3 \cdot b_2 \\
b_4 &= +f(t=-4) - b_0 + 4 \cdot b_1 - 6 \cdot b_2 + 4 \cdot b_3 \\
b_5 &= -f(t=-5) + b_0 - 5 \cdot b_1 + 10 \cdot b_2 - 10 \cdot b_3 + 5 \cdot b_4 \\
b_6 &= -f(t=-6) - b_0 + 6 \cdot b_1 - 15 \cdot b_2 + 20 \cdot b_3 - 15 \cdot b_4 + 6 \cdot b_5
\end{aligned} \qquad (3)$$

2.2 Constants b_{-t} in the A-B Goen Function

The data used to find b_n is a sequence of 13 A Goen and B Goen number data produced from the A-B Goen generating function (Goenawan 2022), namely:

Table 2. Sequence of A and B Goen Numbers

n	t	b_{BG}	b_{AG}
1	0	1	1
2	-1	2	3
3	-2	5	7
4	-3	12	17
5	-4	29	41
6	-5	70	99
7	-6	169	239
8	-7	408	577
9	-8	985	1393
10	-9	2378	3363
11	-10	5741	8119
12	-11	13860	19601
13	-12	33461	47321

2.2.1 AG Function Constants

From table 2 above, a relationship can be obtained to obtain the value of the b_t constant, which is a constituent constant in the IN series (Goenawan 2020). Below in table 3 there are the results of the b_t constant resulting from the A Goen number which is known from table 2.

Table 3. The constant value b_t in the numbers AG

n	t	b_{AG}	b_t	b_t
1	0	1	1	2^0
2	-1	3	-2	-2^1
3	-2	7	2	2^1
4	-3	17	-4	-2^2
5	-4	41	4	2^2
6	-5	99	-8	-2^3
7	-6	239	8	2^3
8	-7	577	-16	-2^4
9	-8	1393	16	2^4
10	-9	3363	-32	-2^5
11	-10	8119	32	2^5
12	-11	19601	-64	-2^6
13	-12	47321	64	2^6

From table 3 above, the AG function as $f_{AG}(n)$ can be used to find the value of the AG number sequence $b_{AG}(n)$ more easily because the constant b_{-t} formed has a regular pattern (Goenawan 2021), namely:

$$b_{-t} = (-1)^{-t} \cdot 2^{roudup(\frac{-t}{2})} \quad (4)$$

2.2.2 BG Function Constants

From table 2 above, a relationship can be obtained to obtain the value of the b_t constant, which is a constituent constant in the IN series (Goenawan 2020). Below in table 4 there are the results of the b_t constant resulting from the B Goen number which is known from table 2.

Table 4. Value of the constant b_t in BG numbers

n	t	b_{BG}	b_t	b_t
1	0	1	1	2^0
2	-1	2	-1	-2^0
3	-2	5	2	2^1
4	-3	12	-2	-2^1
5	-4	29	4	2^2
6	-5	70	-4	-2^2
7	-6	169	8	2^3
8	-7	408	-8	-2^3
9	-8	985	16	2^4
10	-9	2378	-16	-2^4
11	-10	5741	32	2^5
12	-11	13860	-32	-2^5
13	-12	33461	64	2^6

From table 4 above, the BG function as $f_{BG}(n)$ can be used to find the value of the sequence of BG numbers $b_{BG}(n)$ more easily because the b_{-t} constant formed has a regular pattern (Goenawan 2021), namely:

$$b_{-t} = (-1)^{-t} \cdot 2^{rouddown(\frac{-t}{2})} \quad (5)$$

2.3 A-B Goen Function of the IN Series

Because the constant b_t has a regular pattern, the order of the AG and BG numbers can be obtained by using the AG and BG function equations resulting from the Ivan Newton (IN) series. In equation (1), if the value of t is replaced by n, see tables 3 and 4, $t = -(n-1) = -n+1$, then the equation above can be rewritten as:

$$f(n) = b_0 + b_1 \cdot (-n+1) + b_2 \cdot \frac{(-n+1)\cdot(-n+2)}{2!} + \ldots + b_{n-1} \cdot \frac{\prod_{k=1}^{n-1}(-n+k)}{(n-1)!} \quad (6)$$

2.3.1 AG Function

The AG function equation can be obtained with the help of the IN series. The constant b_n is a constituent constant in the IN series contained in equation (4), by replacing the variable $-t$ with $n-1$, it becomes eq.(7):

$$b_n = (-1)^{n-1} \cdot 2^{roudup(\frac{n-1}{2})} \quad (7)$$

by substituting eq.(7) into eq.(6), the AG function equation can be obtained as:

$$f_{AG}(n) = 1 - 2 \cdot (-n+1) + 2 \cdot \frac{(-n+1)\cdot(-n+2)}{2!} + \ldots + (-1)^{n+1} \cdot 2^{roudup(\frac{n-1}{2})} \cdot \frac{\prod_{k=1}^{n-1}(-n+k)}{(n-1)!} \quad (8)$$

So, the value of the AG number sequence is the same as the value produced by the AG function:

$$b_{AG}(n) = f_{AG}(n). \quad (9)$$

2.3.2 BG Function

The BG function equation can be obtained with the help of the IN series. The constant b_n is a constituent constant in the IN series contained in equation (5), by replacing the variable $-t$ with $n-1$, it becomes eq.(10):

$$b_n = (-1)^{n-1} \cdot 2^{rouddown(\frac{n-1}{2})} \quad (10)$$

by substituting eq.(10) into eq.(6), the BG function equation can be obtained as:

$$f_{BG}(n) = 1 - 1 \cdot (-n+1) + 2 \cdot \frac{(-n+1)\cdot(-n+2)}{2!} + \ldots + (-1)^{n+1} \cdot 2^{rouddown(\frac{n-1}{2})} \cdot \frac{\prod_{k=1}^{n-1}(-n+k)}{(n-1)!} \quad (11)$$

So, it can be seen that the value of the BG number sequence is the same as the value produced by the BG function:

$$b_{BG}(n) = f_{BG}(n). \tag{12}$$

The comparison between the AG number and the BG number with the greater the *n* value, the closer the result will be to the root value of two (Goenawan 2022).

III. RESULTS AND DISCUSSION

The A-B Goen generating function is a function that can produce A Goen (AG) numbers and B Goen (BG) numbers by selecting the numbers produced by the generating function with the condition that the numbers only have integer values. Below is the equation for the function $y(t)$ as a generator of integer numbers AG and BG (Goenawan 2022):

$$y(t) = \sqrt{(-1)^t + 2.t^2} \tag{13}$$

where t is a natural number or positive integer, $t = 1, 2, 3, \dots$

Next, a computational comparison is carried out to obtain the value of the A-B Goen number up to the 18th order. From table 5 it can be seen that to find the A-B Goen number with the IN series, you can use the press formulas (8) and (11) with sufficient computational calculations starting from $t = 1$ to steps $t = 18$, whereas if you use the A-B Goen generating function, computational calculations will be required starting from $t = 1$ to $t = 2744210$. To get the same A-B Goen number results, it turns out that the A-B Goen Generating Function (GF) requires greater computational calculations compared to A-B Goen function of the IN Series (INS). If the sequence of A-B Goen numbers being searched for is greater than the computational calculation ratio between GF and INS will be greater as well.

Table 5. Comparison of INS* and GF** computing

No.	$b_{BG}(t)$	$b_{AG}(t)$	INS	GF
1	1	1	1	1
2	2	3	2	2
3	5	7	3	5
4	12	17	4	12
5	29	41	5	29
6	70	99	6	70
7	169	239	7	169
8	408	577	8	408
9	985	1393	9	985
10	2378	3363	10	2378
11	5741	8119	11	5741
12	13860	19601	12	13860
13	33461	47321	13	33461
14	80782	114243	14	80782
15	195025	275807	15	195025
16	470832	665857	16	470832
17	1136689	1607521	17	1136689
18	2744210	3880899	18	2744210

Information:
* is the A-B Goen Function resulting from the IN Series (INS),
** is the A-B Goen (GF) Generating Function.

In table 5 there is a sequence of BG and AG numbers which are the result of dividing the AG and BG numbers in table 6, column 4 is a fractional form of the root value of two. If the sequence of numbers becomes larger, the result will be closer to the root value of two. For the correct comparison of the root value of two to 16 decimal numbers, it is 1.414.213.562.373. 095.

Table 6. Approach $\sqrt{2}$ from dividing A-Goen by B-Goen numbers

No.	$t = b_{BG}(t)$	$b_{AG}(t)$	$b_{AG}(t)/b_{BG}(t)$
1	1	1	1,000.000.000.000.00
2	2	3	1,500.000.000.000.00
3	5	7	1,400.000.000.000.00
4	12	17	1,416.666.666.666.67
5	29	41	1,413.793.103.448.28
6	70	99	1,414.285.714.285.71
7	169	239	1,414.201.183.431.95
8	408	577	1,414.215.686.274.51
9	985	1393	1,414.213.197.969.54
10	2378	3363	1,414.213.624.894.87

11	5741	8119	1,414.213.551.646.05
12	13860	19601	1,414.213.564.213.56
13	33461	47321	1,414.213.562.057.32
14	80782	114243	1,414.213.562.427.27
15	195025	275807	1,414.213.562.363.80
16	470832	665857	1,414.213.562.374.69
17	1136689	1607521	1,414.213.562.372.82
18	2744210	3880899	1,414.213.562.373.14

IV. CONCLUSION

The results of this research further prove the truth of the Order Theory (Goenawan 2021) and the novelty of this research is:

a. Fractions in the numerator and denominator that approach the square root value of two can be generated from the A-B Goen function which is built from the Ivan Newton (IN) series.

b. The A-B Goen numbers obtained from the A-B Goen function with the IN series are computationally more effective and efficient when compared to using the generating function

REFERENCES

Goenawan, Stephanus Ivan. (2020). *Comparison Simulation Analysis Of The Gradual Summation Of A Function With Recognition Of Direct Extrapolation Via IN Series*. IJASST Univ. Sanata Dharma, Yogyakarta.

Goenawan, Stephanus Ivan. (Januari 2022). *Fractional Generating Function from The Square Root of Two with A-B Goen Numbers*, Engineering, Mathematics, and Computer Science (EMACS) Journal, Vol. 4 No. 1. hal 11 – 14, e-ISSN: 2686-2573.

Goenawan, Stephanus Ivan. (Januari 2021). *Order Theory I And II As Foundations For Finding Relationship Between Formulas*, Engineering, Mathematics, and Computer Science (EMACS) Journal, Vol. 2 No. 1. hal 1 – 4, p-ISSN 1410-2765.

Gel'fand, Izrael M.*; Shen, Alexander. (2023). Algebra (8rd ed.). Birkhäuser. p. 120. ISBN 0-8176-3677-3.*

Lord, Nick. (November 2008). "Maths bite: irrational powers of irrational numbers can be rational", *Mathematical Gazette* 92. p. 534.

Marshall, Ash J., and Tan, Yiren. (March 2012). "A rational number of the form a^a with *a* irrational", *Mathematical Gazette* 96. pp. 106-109.

Mitchell, Douglas W. (November 2003). "Using Pythagorean triples to generate square roots of I_2", *Mathematical Gazette* 87. 499–500.

Krantz, Steven George (2006). Calculus: Single Variable, Volume 1. Springer Science & Business Media. p. 248. ISBN 978-1-931914-59-8.

McQuarrie, Donald A. (2003). Mathematical Methods for Scientists and Engineers, University Science Books. ISBN 978-1-891389-24-5

Salas, Saturnino L.; Hille, Einar*; Etgen, Garret J. (2007). Calculus: One and Several Variables (10th ed.).* Wiley. ISBN 978-0-471-69804-3.

Stewart, James (2012). Calculus: Early Transcendentals, 7th ed., Brooks Cole Cengage Learning. ISBN 978-0-538-497 90-9

Thomas, George B., Maurice D. Weir, Joel Hass, Frank R. Giordano (2008), Calculus, 11th ed., Addison-Wesley. ISBN 0-321-48987-X

Zill, Dennis G.; Wright, Scott; Wright, Warren S. (2009). Calculus: Early Transcendentals *(3 ed.). Jones & Bartlett Learning. p. xxvii.* ISBN 978-0-7637-59957.